AS-310: Aircraft Performance
Chapter 1: Basic Physical Concepts

<u>Purpose:</u> Over the years, as we have taught aerodynamics and aircraft performance, we have noticed that students often do not retain certain key information from their introductory physics courses, which makes it difficult for them to do well in aerodynamics and performance. Often, they have not been exposed to aviation related applications of physics, so they do not understand why the subject is important to them. In reality, understanding airplane aerodynamics and performance is ***Applied Physics***. We take certain principles learned from physics and make use of them in order to more fully understand real-world flight related situations. There was a reason to learn physics! Such study was not simply a sophisticated form of mental torture. We want you to know a few fundamentals before we begin our adventures in aerodynamics and performance.

<u>Learning Objectives</u>: After completion of this lesson you should,

1. Understand the four fundamental concepts of "Basic Units", "Basic Equation Basic Units", "Unit Conversion", and "Proportional Thinking".

2. Using the British Gravitational System (BGS), know the definitions and basic units for the most commonly used physical terms which apply to the subject of applied aerodynamics or aircraft performance.

3. Be able to fully explain the key physical principles from introductory physics that apply to the technical understanding of the subjects of aerodynamics and aircraft performance.

<u>Basic Concepts</u>

"<u>**Basic and Derived Units**</u>" The concept of ***Fundamental,*** which we call ***"Basic Units"*** versus ***Derived Units***. is important to grasp if you are going to successfully work problems using physical quantities. You must know the definitions of certain physical quantities and their fundamental or *basic* units. We will use the ***British Gravitational System*** (***BGS***) in this course since it is widely used in aviation. In this system all units are based upon a *distance* unit of the foot, a *force* unit of the pound, and a *time* unit of the second. Many other physical quantities we will use are <u>derived</u> from these three *basic units* (the foot, the pound, and the second).

Table 1.1 on the next page lists the *basic* (fundamental) and *derived mechanical units* one should know for a more advanced study of aircraft performance. The derived units are defined in terms of our three *basic units*, then expressed in British Gravitational System (BGS) units. Next, the symbols used in this book are shown. It is probably worthwhile to take the time to reacquaint yourself with these units. We will be using them quite often in this course.

Quantity (symbol)	**Definition**	**BGS Unit`**	**BGS Symbol**
Fundamental <u>or</u> “Basic Unit”			
Force (F)	Force	pound	p
Length (X)	Length	feet	ft
Time (t)	Time	second	s
Derived Units			
Area (S)	Length ^{2}	feet ^{2}	ft ^{2}
Volume	Length ^{3}	feet ^{3}	ft ^{3}
Velocity (V)	Length / Time	feet / sec	ft / s
Acceleration (a)	Velocity / Time	feet / sec ^{2}	ft / s ^{2}
Mass (m)	Force / Acceleration	pound / feet / sec ^{2} (which is called a *“slug”)*	p / ft / s ^{2} = p-s ^{2} / f = sl
Mass (m)	Weight / Acceleration	pound / feet / sec ^{2}	W / g where, g = 32.174 ft / s ^{2}
Density (ρ)	Mass / Volume	slug / feet ^{3}	p-s ^{2} / f / f ^{3} = sl / ft ^{3}
Work (W)	Force x Distance	pound foot <u>or</u> foot pound	ft-p
Energy (PE or KE)	Force x Distance	pound foot <u>or</u> foot pound	ft-p
Power (P)	Work / Time	foot pound per second	ft-p / s
Momentum (M)	Mass x Velocity	pound second	p-s ^{2} / f x ft / s = p-s
Impulse (I)	Force x Time	pound second	p-s

Table 1.1: <u>Basic and Derived Units - British Gravitational System</u> (BGS)

<u>Example</u> 1.1 Without using Table 1.1, derive the units for impulse?
<u>Hint</u>: Use Newton’s 2nd Law (F = ma)

F = m a = m V / t = M / t (M <u>or</u> m V we call *Momentum*)

s

F t = Impulse = m V = sl (f / s) = (p ~~s~~^{2} / ~~f~~) (~~f~~ / ~~s~~) = <u>p s</u>

"***Basic Equation Basic Units***". When dealing with unfamiliar relationships, it is sometimes difficult to know what units should be used. Many times in numerical calculations keeping track of the proper units is half the battle. This task is easier if we always make it a practice to think in terms of fundamental or *basic units*. If you look at an equation and it does not contain any unusual conversion factors (e.g. no number other than 2, ½, 3, or $^{1}/_{3}$), then you most likely can use any *basic system of units* (British Gravitational or Metric) as long as you are consistent and be relatively comfortable with your results. To solve this sort of *basic equation* simply use *basic units* with your numeric values. You can check this by substituting the units into the equation. The units to the left of the equal sign will exactly equal the units on the right.

Example 1.2 Considering the relationship, F = m a, show that the units on the right side of the equal sign are exactly equal to the units on the left.

Note: This expression does not contain any "unusual" conversion factors; therefore, this is a *basic equation* so *basic units* should work. The "slug" is not the basic unit for mass. The derived unit (slug) should be converted to its basic form ($p\text{-}s^2 / ft$) before continuing.

$$F = m\,a \quad \text{or} \quad p = (sl)\,(ft / s^2) = (p\ \cancel{s}^2 / \cancel{ft})\,(\cancel{ft} / \cancel{s}^2) \quad \text{or} \quad p = p$$

Note: The units on both sides of the equation turn out to be equal. This key point can be helpful to know when trying to figure out what units were used in an equation.

Example 1.3 Consider the *Lift Equation* from Basic Aerodynamics, show that the units on the right side of the equal sign exactly equal the units on the left.

$$V = \sqrt{[295\ W / (C_L\ \sigma\ S)]}$$

Note: Here we see a "strange" number 295. This is a clue to tell us that someone has converted this relationship to some *non-basic* system of units. The *295* is a conversion factor. Unless we derived the equation, we *may not know* the units involved. They could be light years per cubit.

Cannot solve the equation without more information.

"Unit Conversion" Converting from one system of units to another is a lot easier if you are aware of the process also called ***Dimensional Analysis*** (big word describing a simple process). Lets learn what this means by example.

Example 1.4 Consider the situation where you want to convert from knots to feet per second, in order to use a *basic equation* (without a conversion factor).

We can do this by carefully analyzing the units involved.

$$1 \text{ Knot} = \frac{\cancel{\text{Nautical Mile}}}{\cancel{\text{hour}}} \times \frac{6{,}076 \text{ feet}}{\cancel{\text{Nautical Mile}}} \times \frac{1 \cancel{\text{ hour}}}{3{,}600 \text{ seconds}} = \underline{\underline{1.6878 \text{ ft / s}}}$$

Example 1.5 Consider a situation where you want to convert from RPM to radians per second (the *basic unit* for rotational motion is the radian per second or radians / s) in order to use a *basic equation* without a conversion factor.

$$1 \text{ RPM} = \frac{\cancel{\text{Revolution}}}{\cancel{\text{Minute}}} \times \frac{2\pi \text{ Radians}}{\cancel{\text{Revolution}}} \times \frac{1 \cancel{\text{ Minute}}}{60 \text{ Seconds}} = \underline{\underline{0.10472 \text{ radians / s}}}$$

Next is an example of using this conversion factor,

Example 1.6 Convert 2,400 RPM to "basic" physical units.

$$2{,}400 \text{ Rev / min} \times 0.10472 \text{ radians / sec / min} = \underline{\underline{251 \text{ radians / s}}}$$

"**Proportional Thinking**" Take the expression for kinetic energy, $KE = \frac{1}{2} m V^2$. This expression states that KE is directly proportional to mass (m) and velocity (V^2) or $KE \propto m$ and $KE \propto V^2$. The symbol "$\propto$" means "is proportional to". If either mass or velocity are increased, the kinetic energy (KE) will increase. This type of relationship is called a ***Direct Proportion***. If only one factor in this expression is varied at a time, the direct proportion can be expressed as follows,

$$KE = K_1 m, \quad \text{where } K_1 = \tfrac{1}{2} V^2 \quad \underline{\text{or}} \quad KE = K_2 V^2, \quad \text{where } K_2 = \tfrac{1}{2} m$$

Basic Concepts

"Proportional Thinking"

Varying one factor at a time is a handy procedure called completing a ***Parametric Study***. This sort of study is often used to examine the impact of one factor in a relationship on the resulting answer. In this case, how much will kinetic energy (KE) change if only mass (m) is increased or how much will kinetic energy change if only velocity (V) is increased?

The expression $KE = K_1 m$ can be rewritten as, $KE / m = K_1$ (a constant)

Comparing two situations, Case 1 where $m = m_1$ and Case 2 where $m = m_2$

$KE_1 / m_1 = KE_2 / m_2$ or $KE_2 = KE_2 (m_2 / m_1)$

The given direct proportion has now been expressed as a ***Ratio***. See the following examples for clarification of the importance of *proportional thinking*.

Example 1.7 Say that an airplane's mass (m) is increased by 50%. By how much will its kinetic energy (KE) change?

In this case, $m_2 = 1.5\ m_1$ (Representing a 50% increase in mass)

Expressing the fact that $KE \propto m$ (is proportional to) m, as a *ratio*,

$$KE_2 = KE_1 (m_2 / m_1) = KE_1 (1.5 \cancel{m_1} / \cancel{m_1})$$

It can now be said that,

$KE_2 = 1.5\ KE_1$ or Kinetic energy has increased by 50%

Example 1.8 Say that a airplane's velocity (V) is increased by 50%. By how much will the kinetic energy (KE) change?

In this case, $V_2 = 1.5\ V_1$ (Representing a 50% increase in velocity)

Using the fact that $KE \propto V^2$ (is proportional to) V^2 and expressing it as a *ratio*,

$$KE_2 = KE_1 (V_2 / V_1)^2 = KE_1 (1.5 \cancel{V_1} / \cancel{V_1})^2$$

Therefore,

$KE_2 = 2.25\ KE_1$ or Kinetic energy has increased by 125%
(2.25 times KE_1 is a 125% increase)

Basic Concepts

"Proportional Thinking"

Example 1.8 (continued)

Note Do you understand why increasing velocity (V) has a greater impact on kinetic energy (KE) than increasing the mass (m)?

Consider the uniformly accelerated rectilinear motion expression relating time to distance and acceleration, $t = \sqrt{(2X/a)}$. In this case t is *directly proportional* to $\sqrt{X}$ and *inversely proportional* to $\sqrt{a}$. If distance (X) increases, the time (t) will increase but if acceleration (a) increases, the time (t) will decrease. This second type of relationship is called an ***Inverse Proportion***. Again, if only one factor is varied at a time, these proportions can be expressed as follows,

$t = \sqrt{(2X/a)}$ or $K_1 \sqrt{X}$, where $K_1 = \sqrt{(2/a)}$ (Direct proportion)

and

$t = \sqrt{(2X/a)}$ or $K_2 (1/\sqrt{a})$, where $K_2 = \sqrt{(2X)}$ (Inverse proportion)

Again, varying one factor at a time a *parametric study* is performed. This sort of study can again be used to examine the impact of varying one factor on the resulting answer. In this case, focusing on the *inverse relationship*, how much will time (t) change if acceleration (a) is increased?

The expression $t = K_2 (1/\sqrt{a})$ can be rewritten as, $t\sqrt{a} = K_2$ (a constant)

Comparing two situations, Case 1 where $a = a_1$ and Case 2 where $a = a_2$

$$t_1 \sqrt{a_1} = t_2 \sqrt{a_2}$$

The given *inverse proportion* can also be expressed as a *ratio*,

$$t_2 = t_1 \sqrt{(a_1 / a_2)}$$

"Proportional Thinking"

Example 1.9 An airplane is experiencing uniformly accelerated rectilinear motion. By how much will the time (t) change for the airplane to move a given distance (X) if acceleration (a) is increased by thirty (30) percent?

In this case, $a_2 = 1.3\, a_1$ (Representing a 30% increase in acceleration)

Expressing the fact that $t \propto 1/a$ (is proportional to) $1/a$ as a *ratio*,

$$t_2 = t_1 (a_1 / a_2) = t_1 (\cancel{a_1} / 1.3\, \cancel{a_1})$$

It can now be said that,

$$t_2 = t_1 / 1.3 = \underline{0.877\, t_1} \quad \text{or} \quad \underline{\text{Time has decreased by 12.3 \%}}$$

Example 1.10 Repeat Example 1.9 except decrease acceleration (a) by thirty (30) percent.

In this case, $a_2 = 1 - 0.3 = 0.7\, a_1$ (A 30 % decrease in acceleration)

Again, expressing the fact that $t \propto 1/a$ (is proportional to) $1/a$ as a *ratio*,

$$t_2 = t_1 (a_1 / a_2) = t_1 (\cancel{a_1} / 0.7\, \cancel{a_1})$$

It can now be said that,

$$t_2 = \underline{1.195\, t_1} \quad \text{or} \quad \underline{\text{Time has increased by 19.5\%}}$$

Important Physical Principles

***Newton's First Law* (The Law of Inertia)** A body left to itself has constant velocity (speed and direction).

***Newton's Second Law* (The Law of Momentum)** The acceleration (a) of a body is directly proportional to the resultant force (F) acting on the body, is inversely proportional to the mass (m) of the body, and has the same direction as the resultant force (F). The resultant force (F) acting on a body can be the vector sum of the many forces.

Re-arranging the terms slightly, this principle can be expresses as,

1.1	$F = m a$	(Newton's Second Law - also called Newton's Law of Momentum)

Where, F = force (p)
m = mass (sl)
a = acceleration (ft / s^{2})

Note: To change something's velocity we must apply a force.

Example 1.11 What would be the acceleration on a 130,000 pound airplane subject to a net unbalanced force of 30,000 pounds?

Using Newton's Second Law (Equation 1.1), $F = m a$

$a = F / m$ Where, $m = 130{,}000 \text{ p} / 32.174 \text{ ft} / s^2 = 4040.5 \text{ sl}$

$a = 30{,}000 \text{ p} / 4040.5 \text{ p-}s^2 / \text{ft} = \underline{\underline{7.4 \text{ ft} / s^2}}$

Note: In this example the slug was expressed in it's "basic" form, p-s^2 / ft. In this way the units work out for acceleration as ft / s^2.

Important Physical Principles

***Newton's Second Law* (Newton's Law of Momentum)**

Sometimes Newton's Second Law is expressed as the ***Time Rate of Change of Momentum***. We know that applying a force for a given amount of time will change an object's momentum.

$$F = m a = m (V / t) = \frac{mV}{t} = M / t \qquad \text{Where, } mV \text{ or momentum (M)}$$

1.2 F = M / t or F t = M , Where F t = I or Impulse

Where,

	F	= force (p)
	M	= momentum (p-s)
	t	= time (s)
	I	= impulse (p-s)

Note: *F t* is defined as *impulse (I)*. To change something's momentum (M) we must apply an impulse (I) or a force (F) for a given amount of time (t).

Example 1.12 What impulse (I) is required to change an airplane's momentum (M) by 200,000 p-s?

Applying Newton's Second Law directly, we know that momentum is changed by applying an impulse,

F t = Momentum (M) = Impulse (I) = 200,000 p-s

Example 1.13 How long would a 20,000 pound force need to be applied in order to change an airplane's momentum by 200,000 p-s?

Using Newton's Second Law in terms of momentum (Equation 1.2),

F t = M, solving for *t*

t = M / F = 200,000 p-s / 20,000 p = 10 s

Important Physical Principles

Newton's Third Law ***(The Law of Reaction)***. For every action there is an equal and opposite reaction. This principle is the basis for airplane propulsion. Mass that is accelerated rearward behind the airplane results in an equal but opposite force we call thrust. Thrust is what propels an airplane through the air. The airplane's motion in the same direction as its thrust.

Mechanical Energy: There are two types of mechanical energy that are helpful to know about when examining aircraft performance: potential energy and kinetic energy. ***Potential Energy*** is the energy an object possesses based upon its position. ***Kinetic Energy*** is the energy an object possesses due to its motion.

Potential energy (PE) can be expressed as an objects weight (W) times its height (H). We know that the force, W = m g,

1.3 PE = W H or m g H

$$PE = W\,H \quad \text{or} \quad m\,g\,H \qquad (1.3)$$

Where,
- PE = potential energy (ft-p)
- W = weight (p)
- H = height (ft)
- m = mass (slugs)
- g = 32.174 ft / s ^{2} = constant in the BGS

Closely examining the units in terms of "basic units",

$$PE = \frac{(\text{pound} \times \cancel{\text{second}}^2)}{\cancel{\text{foot}}} \times \frac{\cancel{\text{foot}}}{\cancel{\text{second}}^2} \times \text{foot} = \text{foot x pounds} = \underline{\text{ft\_p}}$$

Note: The ft-p unit is the same as we defined earlier in general for energy
. Review Table 1.1 if need be.

Example 1.14 What is the potential energy (PE) of a 193 pound sky diver falling from 1000 feet above the ground?

Using the definition of potential energy (Equation 1.3), PE = W H

The sky diver weighs 193 p (givrn)

PE = 193 p (1,000 ft) = 193,000 ft-p

Important Physical Principles

Mechanical Energy

An object's kinetic energy (KE) can be expressed as,

1.4 $KE = \frac{1}{2} m V^2$

Where,

KE	= kinetic energy (ft-p)
m	= mass (sl)
V	= velocity (ft / s)

Ignoring the ½ for now since we are only talking about the *units* involved,

$$KE = \frac{1}{2} m V^2 = sl\,(ft / s)^2$$

$$KE = \frac{(p\text{-}\cancel{s}^2)}{\cancel{ft}} \times \frac{\cancel{ft}}{\cancel{s}} \times \frac{ft}{\cancel{s}} = \underline{ft\text{-}p}$$

Example 1.15 How much kinetic energy (KE) must the brakes of a 400,000 p Boeing 747 absorb while stopping an airplane after touching down at 150 ft / s? You will see this is a LOT of energy.

Using Equation 1.4, $KE = \frac{1}{2} m V^2$

$m = (400{,}000\ p) / (32.174\ ft / s^2) = 12{,}432\ p\ s^2 / ft$ or 12,432 sl

$KE = \frac{1}{2} m V^2 = \frac{1}{2} (12{,}432\ p\ s^2 / ft)(150\ ft / s)^2 = \underline{140\ \text{million ft-p}}$

Note: 140 million ft-p of mechanical energy converts to a lot of heat in the brakes. Remember, $p\ s^2 / ft$ = *slug* (sl) by definition

Conservation of Mechanical Energy. Energy can be neither created nor destroyed, but it may change form. One important application of this concept is with ***Total Mechanical Energy (TE)***,

Total Mechanical Energy (TE) = Potential Energy (PE) + Kinetic Energy (KE),

1.5 $TE = PE + KE = m g H + \frac{1}{2} m V^2$

Where,

TE	= total energy (ft-p)	m	= mass ($p\text{-}s^2 / ft$ or slug)
PE	= potential energy (ft-p)	g	= $32.174\ ft / s^2$
KE	= kinetic energy (ft-p)	V	= velocity (ft / s)

Important Physical Principles

Mechanical Energy

Example 1.16 How fast will the sky diver in Example 1.14 hit the ground if his or her chute does not open? Assumption: Ignore the effects of drag.

From Example 1.14 we see that the sky diver will trade 193,000 ft-p of potential energy for an equal amount of kinetic energy during his or her fall. If the change of potential energy (reduction) equals the change in kinetic energy (increase) the impact velocity may be determined..

Δ PE = Δ KE

$\Delta PE = \frac{1}{2} m V_I^2 - \frac{1}{2} m V_F^2$ (Equation 1.3 and 1.4)

$\Delta PE = \frac{1}{2} (0)^2 - \frac{1}{2} m V_F^2 = 193{,}000$ ft-p from Example1.14

We will say that the potential energy is negative, since it is reducing,

$m = (193 \text{ p}) / (32.174 \text{ ft} / s^2) = 5.9986 \text{ p-s}^2 / \text{ft}$ or 5.9986 sl

$-193{,}000 \text{ ft-p} = -\frac{1}{2} (5.9986 \text{ p-s}^2/\text{ ft}) V_F^2 = -\frac{1}{2} (5.9986 \text{ p-s}^2/\text{ ft}) V_F^2$

or, $V_{Final} = \sqrt{} [(193{,}000 \text{ p}) (2)] / (5.9986 \text{ p-s}^2 / \text{ft})$ = 254 ft / s

Note: 254 ft / s is approximately 150 knots or 173 mph! The existence of aerodynamic drag will actually limit the speed to a "terminal velocity" of approximately 120 mph or 104 knots). Here we ignored drag.

Continuity Principle. This concept implies that the mass of a fluid (e.g. air) flowing through a constriction (or over a wing) must be constant as the area of the constriction changes. In the case of compressible flow or flow where density changes,

1.6	$\rho A V$	= Constant	(Compressible Flow)

Where,
ρ = density (sl / ft ^{3})
A = area (ft ^{2})
V = velocity (ft / s)

Checking the units carefully,

(sl / ~~ft~~ ^{3}) (~~ft~~ ^{2}) (~~ft~~ / s) = sl / s = Constant

We call airflow of a given amount of slugs per second, ***Mass Flow***.

Important Physical Principles

Continuity Principle

Example 1.17 The Wrong Brothers were designing a wind tunnel in order to test a model of their soon to be famous Wrong Flyer. They wanted a test section flow velocity of 50 knots but their wind tunnel fan could only produce a flow of 25 knots. Considering the wind tunnel as a simple venturi of gradually reducing area, how small must the test section area have to be to make this wind tunnel accomplish their goal? The area of the fan is 30 ft ^{2}.

Using the principle of continuity (Equation 1.6),

ρ A V = constant — Where, we may assume ρ is also constant since the flow will no doubt be incompressible at these slow speeds.

$$\rho \; A_{Fan} \, V_{Fan} = \rho \; A_{Test\ Section} \, V_{Test\ Section} = \text{Constant}$$

$$A_{Fan} \, V_{Fan} = A_{Test\ Section} \, V_{Test\ Section}$$

$$(30\ \text{ft}^2)\,(25\ \cancel{\text{knots}}) = A_{Test\ Section}\,(50\ \cancel{\text{knots}})$$

or

$$A_{Test\ Section} = \underline{\underline{15\ \text{ft}^2}}$$

Note: In this case it was unnecessary to convert to ft / s since the knot unit is on both sides of the equation and therefore cancels out

When density (ρ) is constant as in the case of incompressible flow Equation 1.6 may be rewritten as,

1.7	A V = Constant		(Incompressible Flow)

Where,
- A = area (ft ^{2})
- V = velocity (ft / s)

Note: As in the previous example any unit for velocity may be used in this relationship. For example, you may use knots directly.

Important Physical Principles

Uniformly Accelerated Rectilinear Motion. Motion in a straight line with constant acceleration occurs often enough in aviation for it to be important to us. For example, drop your beer mug and it heads for the floor (or you foot) in a straight line accelerating at a constant 32.174 ft / s ^{2}. We certainly could use the *Principle of Conservation of Energy* discussed previously to find out how fast the beer mug will hit your foot. We can also solve this dilemma recognizing that this situation can be described as *Uniformly Accelerated Rectilinear Motion* (motion in a straight line). Three handy relationships describe this type of motion.

We know that acceleration equals velocity divided by time or $a = \Delta V / t$

$\Delta V = a\,t$ Where, ΔV is a change in velocity

Comparing two different velocities, one before accelerating (V_I) and one after accelerating (V_F) and assuming that acceleration (a) is constant,

1.8 $V_F = V_I + \Delta V = V_I + a\,t$ (Relates V, a, and t)

Where,
- V_F = final velocity (ft / s)
- V_I = initial velocity (ft / s)
- ΔV = change in velocity (ft / s)
- a = acceleration (ft / s ^{2})
- t = time (s)

Knowing that Distance = V_{AVE} x Time (We will say Distance = X)

$$X = V_{AVE} \times \text{Time} = \tfrac{1}{2}(V_I + (V_I + a\,t))\,t = (V_I + \tfrac{1}{2}a\,t)\,t$$

1.9 $X = V_I\,t + \tfrac{1}{2}a\,t^2$ (Relates V, a, t, and X)

Where,
- X = distance (ft)
- V_I = initial velocity (ft / s)
- a = acceleration (ft / s ^{2})
- t = time (s)

Important Physical Principles

Uniformly Accelerated Rectilinear Motion

Using Equation 1.8, $V_F = V_I + a\,t$ and solving for t,

$$t = (V_F - V_I) / a$$

Substituting into Equation 1.9, $X = V_I\,t + ½\,a\,t^2$

$$X = V_I\,[(V_F - V_I) / a] + ½\,a\,[(V_F - V_I) / a]^2$$

Re-arranging the terms,

1.10	$V_F^{\,2} = V_I^{\,2} + 2\,a\,X$	(Relates V, a, and X)

Where V_F = final velocity (ft / s)
V_I = initial velocity (ft / s)
a = acceleration (ft / s $^{2)}$
X = distance (ft)

Example 1.18 What would the takeoff distance to be for a 300,000 p Boeing 767 if the airplane accelerates at a constant 7 ft / s ^{2}? Assume that the lift off speed is 150 knots.

Recognizing this as an example of *Uniformly Accelerated Rectilinear Motion,* we can apply Equation 1.10. We must be careful since the speed is given in knots. The knot is not a basic unit (perhaps a pun?).

$V_F^{\,2} = V_I^{\,2} + 2\,a\,X$ and $V_I = 0$ ft /s

$V_F = (150\ \text{~~knots~~})\ ((1.6878\ \text{ft / s /}\ \text{~~knot~~}\ ) = 253.17$ ft / s

$(253.17\ \text{ft / s})^2 = (0\ \text{ft / s})^2 + 2\,(7\ \text{ft / s}^2)\,X$ ft

Solving for X, X = 4,578 ft.

Example 1.19 Complete Example 1.16 (sky diver problem) using the principle of uniformly accelerated rectangular motion. Compare your answers.

Equation 1.10 states, $V_F^{\,2} = V_I^{\,2} + 2\,a\,X$ where in this case "a" is the acceleration of gravity (32.174 ft / s ^{2})

$V_F^{\,2} = 0 + 2\,(32.174\ \text{ft / s}^s)\,(1{,}000\ \text{ft}) = 64{,}348\ \text{ft}^2 / \text{s}^2$

Example 1.19 (continued)

Solving for $V_F = \sqrt{(64,348 \text{ ft}^2 / \text{s}^2)} \quad = \quad \underline{254 \text{ ft / s}}$

Note: This is the same answer as found previously using the principle of conservation of mechanical energy. See Example 1.16.

Example 1.20 Given the information shown below, determine how fast should this airplane should be going when it passes the 2000-foot runway marker (2000 feet from the start of the takeoff roll)? Please express your answer in knots.

GIVEN INFORMATION

Gross weight	=	130,000 p
Average Thrust	=	50,000 p
Average friction	=	1,300 p
Average Drag	=	6,500 p
V_{LOF}	=	140 knots

The net force (Σ F) accelerating the airplane is,

$$\Sigma F = \text{Thrust - drag - friction}$$

$$\Sigma F = 50,000 \text{ p} - 6,500 \text{ p} - 1,300 \text{ p} = 42,200 \text{ p}$$

This value represents the *net force* that accelerates the airplane.

The airplane's mass (m) is,

$$m = W / g = 130,000 \text{ p} / 32.174 \text{ ft / s}^2 = 4040.5 \text{ p-s}^2 / \text{ft or slugs}$$

From Newton's Second Law, the airplane's acceleration (a) is,

$$a = \Sigma F / m = (42,200 \text{ p}) / (4,040.5 \text{ p-s}^2 / \text{ft}) = 10.444 \text{ ft / s}^2$$

Using Equation 1.10 or $V_F{}^2 = V_I{}^2 + 2 a X$,

$$V_F{}^2 = 0^2 + 2 (10.444 \text{ ft / s}^2)(2,000 \text{ ft}) = 41,776 \text{ ft}^2 / \text{s}^2$$

Solving for takeoff distance (V_F),

$$V_F = \sqrt{(41,776 \text{ ft}^2 / \text{s}^2)} = \underline{204.39 \text{ ft / s}}$$

$$V_F = (204.39 \text{ ft / s}) / (1.6878 \text{ ft /s / knot}) = \underline{121 \text{ knots}}$$

Important Physical Principles

Friction. Frictional force (*f*) is related to the ***Normal Force (N)*** (force perpendicular to a surface) and the coefficient of friction (μ) . It is not related to the amount of contact surface area. The ***Rolling Friction Coefficient*** of friction of a tire is normally assumed as 0.02 while the braking coefficient is normally assumed equal to 0.6 to 0.8, depending upon the type of tire used and the condition of the runway. The actual ***Braking Friction Coefficient*** varies with the percent tire slip allowed. ***Anti-Skid Braking Systems*** attempt to maximize braking by allowing the optimum amount of slip. This topic will be discussed in more depth in Chapter 8 - Aborted Takeoff.

1.11	$f = \mu N$

Where,
- f = frictional force (p)
- μ = coefficient of friction (see Table 1.2 for braking coefficient)
- N = the force of a tire on the runway (called the "normal force") This normal force will normally equal the weight of the airplane minus the amount of lift the wings are producing.

Note: The normal force may also depend upon runway slope. We will explore that issue later.

Table 1.2: **Estimated** Runway Friction Coefficients
(Reference Aircraft Performance by Layton)

Runway Surface	Friction Coefficient (μ)
Rolling Friction	
Dry concrete / asphalt	0.02
Hard turf	0.04
Short grass	0.05
Long grass	0.05
Soft ground	0.10 to 0.30
Braking friction	
Dry concrete / asphalt	0.75
Damp concrete	0.75
Light rain, concrete	0.60
Heavy rain, concrete	0.50
10 % snow residue	0.40
5 inches of light snow	0.37
Heavy rain, asphalt	0.25
12 ^{0}F ice, asphalt	0.19
32 ^{0}F ice, asphalt	0.15

Important Physical Principles

Friction

Example1.21 An aircraft weighs 130,000 p and has 80 percent of its weight on the main (braking) wheels. The runway is dry concrete or asphalt. Find the braking force (*f*) on the airplane. Assume that there are no brakes on the nose wheel.

The weight on the main (braking) wheels is,

$$0.80 \times 130{,}000 \text{ p} = 104{,}000 \text{ p}$$

Using Equation 1.11, $f = \mu N$ and Table 1.2, the frictional force (f) is,

$$f = 0.75\,(104{,}000 \text{ p}) = \underline{\underline{78{,}000 \text{ p}}}$$

Braking friction coefficient on dry concrete is 0.75 (Table 1.2)

Example 1.22 By how much would the braking force decrease if there was 10% snow residue on the runway?

Using Equation 1.11 and Table 1.2 again, the new frictional force (f) is,

$$f = 0.40\,(104{,}000) = 41{,}600 \text{ p}$$

Braking friction coefficient on 10% snow residue is 0.40 (Table 1.2)

The frictional force has decreased significantly,

$$f = 78{,}000 \text{ p} - 41{,}600 \text{ p} = \underline{\underline{34{,}125 \text{ p}}}$$

Note: This is a very significant decrease in braking force which would result in almost twice the stopping distance!

Example 1.23 Assuming there is no aerodynamic drag and no reverse thrust is used, determine the minimum possible stopping distance of the airplane in Example 1.21. Say that the airplane touches down at 100 knots and maximum braking is applied from touchdown to stop.

The airplane's mass (m) is,

$$m = W / g = (130{,}000 \text{ p}) / (32.174 \text{ ft} / \text{s}^2) = 4040.3 \text{ p-s}^2 / \text{ft}$$

(or 4,040.3 sl)

Friction

Example 1.23 (continued)

The net force slowing the airplane (Σ F) is,

$$\Sigma F = \text{Braking force} = -78{,}000 \text{ p}$$

("-" means the force slows the airplane)

The acceleration is,

$$a = \Sigma F / m = (-78{,}000 \text{ p}) / (4040.5 \text{ p-s}^2 / \text{ft}) = -19.3 \text{ ft} / \text{s}^2$$

(decelerating)

Using Equation, 1.10: $V_F^{\,2} = V_I^{\,2} + 2 a X$,

V_I = 100 knots and V_F = 0 knots

$$0^2 = (100 \text{ knot x } 1.6878 \text{ ft / s / knot})^2 + 2(-19.3 \text{ ft / s}^2) X \text{ ft}$$

or X = 738 ft.

Note This answer seems to be a very short distance! Remember, the assumption was made that the brakes were slammed on exactly at touchdown. Full braking was applied until the airplane stops. Both of these actions would be inappropriate from a pilot technique or passenger comfort viewpoint. Also, this distance represents only the ground stopping distance. No allowance is added for descent from the beginning of the runway to touchdown. Actual stopping distance will be significantly longer than this *idealized* result. More realistic stopping distances will be better determined later.

Terms or Concepts You Should Know :

Acceleration
Anti-Skid Braking System
Applied Physics
Area
* “Basic Units”
* “Basic Equation Basic Units”
* “Basic Conversion”
Braking Friction Coefficient
British Gravitational System
Conservation of Mechanical Energy
Continuity Principle
Density
Derived Units
Dimensional Analysis
Direct Proportion
Energy
Force
Friction
Horse Power
Impulse
Inverse Proportion
Kinetic energy
Length
Mass
Mechanical Energy
Momentum
Newton’s First Law
Newton’s Second Law
Newton’s Third Law
Normal Force
Potential Energy
Power
Rolling Friction Coefficient
* ”Proportional Thinking”
Slug
The Law of Inertia
The Law of Momentum
The Law of Reaction
Time
Total Mechanical Energy
Uniformly Accelerated Rectilinear Motion
Volume
Work

Note: * Unique Author Defined Terms

Useful Equations:

* Key Relationships you should KNOW

*	1.1	$F = m a$	(Newton's Second Law - also called The Law of Momentum)
	1.2	$F = M / t$ or $F t = M$, Where $F t = I$ or Impulse	
	1.3	$PE = W H$ or $PE = m g H$	(Mechanical Potential Energy)
	1.4	$KE = ½ m V^2$	(Mechanical Kinetic Energy)
*	1.5	$TE = PE + KE = m g H + ½ m V^2$	(Conservation of Energy)
*	1.6	$\rho A V = \text{Constant}$	(Continuity - Compressible)
	1.7	$A V = \text{Constant}$	(Continuity - Incompressible)
*	1.8	$V_F = V_I + \Delta V = V_I + a t$	(Relates V, t, and a)
	1.9	$X = V_I t + ½ a t^2$	(Relates X, V, a, and t)
*	1.10	$V_F^2 = V_I^2 + 2 a X$	(Relates V, X, and a)
	1.11	$f = \mu N$	(Frictional Force)

Useful Constants or Conversion Factors

g	=	32.174 ft / s ^{2}	1 knot	=	1.6878 ft / s
1 hp	=	550 ft-p / s	1 RPM	=	0.10472 1 / s (or radians / s)
			1 sm / hr	=	0.867 nm / hr

Symbology

P	=	horsepower (ft-p / s)
T	=	thrust (p)
V	=	velocity (ft / s)
F	=	net accelerating force (p)
m	=	mass (sl <u>or</u> p-s ^{2} / ft)
a	=	acceleration (ft / s ^{2})
M	=	momentum (p-s)
I	=	impulse (p-s)
t	=	time (s)
PE	=	potential energy (ft-p)
KE	=	kinetic energy (ft-p)
TE	=	total energy (ft-p)
ρ	=	density (sl / ft ^{3} <u>or</u> p-s ^{2} / ft / ft ^{3})
A	=	cross sectional area (ft ^{2})
V_I	=	initial velocity (ft / s)
V_F	=	final velocity (ft / s)
X	=	distance (ft)
f	=	frictional force (p)
μ	=	coefficient of friction
N	=	normal force (p)
ΔV	=	change in velocity (ft / s)
α	=	means "is proportional to"

Related Homework :

"Basic and Derived Units" - British Gravitational System (BGS)

For Questions 1.1 to 1.14, what are the physical units in the British Gravitational System for the following,

1.1. Length	1.2. Mass	1.3. Time	1.4. Area
1.5. Volume	1.6. Speed	1.7. Acceleration	1.8. Force
1.9. Density	1.10. Energy	1.11. Power	1.12. Work
1.13. Momentum	1.14. Impulse		

"Basic Equation Basic Units"

1.15 Express the principle "Basic Equation Basic Units" in your own words.

"Unit Conversion"

1.16 Consider the expression $q = ½ \rho V^2$ (dynamic pressure equals ½ x density x velocity squared). Show that the units on the right side of the equal sign are exactly equal to the units on the left.

1.17. Derive a conversion factor to convert knots to feet per second.

1.18. Derive a conversion factor to convert revolutions per minute to radians per second.

1.19 Derive a conversion factor to convert statute miles per hour to knots.

The Concept of "Proportional Thinking"

1.20 If dynamic pressure is proportional to the square of velocity ($q \propto V^2$), what happens to the dynamic pressure on an object if its velocity is decreased by thirty percent (30%)?

<u>NOTE</u>: This question introduces a <u>key concept</u> - working with proportions to estimate changes in a relationship. This skill will have <u>much applicability</u> later.

Related Homework :

Mechanical Energy

1.21 What is the kinetic energy of a 50,000 p. (earth weight) lunar module impacting the moon at 1,000 ft / s? Assume that the moon's gravity is one sixth that of the earth.

1.22 An object of a given mass moving the same velocity will possess the same kinetic energy crashing into Pluto as it would crashing into the earth. Note: Pluto has far less gravity than earth. (A = True or B = False)

GIVEN INFORMATION: (Problems 1.23 to 1.27)

Aircraft Gross Weight	= 16,200 p.
Altitude	= 5,000 ft.
Speed	= 120 knots

1.23 Find the potential energy.

1.24 Find the kinetic energy.

1.25 Find the total energy.

1.26 Assuming no extra drag on the airplane, if the pilot dove until his/her speed was 240 knots, what would the altitude be?

1.27 Assuming conservation of energy, when an aircraft trades off altitude for a gain in speed _____________ is decreased.

A. Kinetic energy B. Total energy C. Potential energy

Continuity Principle

1.28 After crashing their first six airplanes, the Wrong Brothers re-designed their wind tunnel in order to test a new model of their soon to be famous Wrong Flyer II. They wanted a test section flow velocity of 65 knots but their wind tunnel fan could still only produce a flow of 25 knots. Considering the wind tunnel as a simple venturi of gradually reducing area, how small must the test section area have to be to accomplish their goal, if the area of the fan is 30 ft^2?

Related Homework :

Uniformly Accelerated Rectilinear Motion and/or Newton's Law of Momentum

GIVEN INFORMATION: (Questions 1.29 to 1.32)

Gross Weight	=	300,000 p
Average Drag	=	15,000 p
Average Friction Force	=	3,000 p
Average Thrust	=	80,000 p
Lift Off Speed	=	140 KTAS

1.29 Compute the acceleration on the aircraft during the takeoff roll (ft / s^2).

1.30 What would be the length of the takeoff run (ft)?

1.31 How long would it take until liftoff once the takeoff roll is started (s)?

1.32 Given the information shown above, determine how fast should this airplane should be going when it passes the 2,000-ft runway marker (2,000 ft from the start of the takeoff roll)? Please express your answer in knots.

Friction

1.33 An aircraft weighs 24,000 lbs. and has 75 percent of its weight on the main (braking) wheels. If the coefficient of braking friction is 0.70, find the braking force on the airplane. Assume that there are no brakes on the nose wheel.

1.34 An aircraft weighs 300,000 lbs. and has 70 percent of its weight on the main (braking) wheels. The runway is damp concrete. Find the braking force on the airplane. Assume that there are no brakes on the nose wheel.

Selected Answers:

1.1 ft	**1.2 (p-s ^{2}) / ft	1.3 s
1.4 ft ^{2}	1.5 ft ^{3}	1.6 ft / s
1.7 ft / s ^{2}	1.8 p	**1.9 ((p-s ^{2}) / ft) / ft ^{3}
1.10 ft-p	1.11 ft-p / s	1.12 ft-p
1.13 p-s	1.14 p-s	1.15 Use your own words
1.16 Substitute basic units into relationship		1.17 1 k = 1.6878 ft / s
1.18 1 rpm = 0.1047 rad / s		1.19 1 sm/h = 0.869 knot
1.20 $q_2 = 0.49\ q_1$ (a 51 % decrease)	1.21 7.77 (10 ^{8}) ft-p	1.22 A
1.23 81 (10 ^{6}) ft-p	1.24 10.3 (10 ^{6}) ft-p	1.25 91.3 (10 ^{6}) ft-p
1.26 3,089 ft	1.27 C	1.28 11.5 ft ^{2}
1.29 6.65 ft / s ^{2}	1.30 4198 ft	1.31 35.5 s
1.32 163 f / s or 97 knots	1.33 12,600 p	1.34 157,500 p

** Note: We normally call a p-s ^{2} / ft a "Slug" or "sl ". The slug is a derived unit. If allowed to use derived units, Question 1.2 could be answered "sl" and Question 1.9 could be answered "sl / ft ^{3}".